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SE-6165

B. E. II (Sem. III) (Comp. I.T.) Examination

May / June – 2011

Discrete Mathematics

Time : **3** Hours] [Total Marks : **100**

Instructions :

(1)

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fillup strictly the details of ☛ signs on your answer book.

Name of the Examination :
☛ B. E. II (Sem - III) (Comp. I.T.)

Name of the Subject :
☛ Discrete Mathematics

☛ Subject Code No. : 6 1 6 5 ☛ Section No. (1, 2,.....) : Nil

Seat No. :

Student's Signature

(2) Attempt all questions.

(3) Figure to the right indicate marks.

1 (a) Do as directed. **10**

(1) Define union, intersection and ring-sum of two graphs.

(2) Define isomorphism.

(3) Define component.

(4) Define deletion of a vertex and deletion of an edge.

(5) Define (i) Path (ii) Circuit.

(b) Attempt any two : **10**

(1) Prove that a simple graph with n vertices and k components can have atmost $\frac{(n-k)(n-K+1)}{2}$ edges.

(2) Prove that if there are n vertices in a graph and e edges then $\sum_{i=1}^{n} d(\vartheta_i)=2e$, where V_i devotes the vertices.

(3) Prove that an infinite graph with finite number of vertices must have atmost one pair of vertices joined by infinite number of parallel edges.

2 (a) Describe Konigseberg bridge problem. **03**

(b) Attempt any three : **12**

(1) Define a binary tree. Show that the number of vertices in a binary tree is always odd. Obtain number of pendant vertices in a binary tree.

(2) Prove that a graph is a tree if and only if it is minimally connected.

(3) Define a tree. Prove that a tree with n vertices has n-1 edges.

(4) Prove that a given connected graph G is an Euler graph, if and only if all vertices of G are of even degree.

3 (a) Describe three-utilities problem **03**

(b) Attempt any three : **12**

(1) In any simple, connected plannar graph with f regions, n vertices and e edges (e>2) show that

$e \geq \frac{3}{2} f$ and $e \leq 3n-6$

(2) Define a cut-set. Prove that every circuit has an even number of edges in common with any cut-set.

(3) Define a plannar graph. Prove that a connected plannar graph with n vertices and e edges has e-n+z regions.

(4) Define a spanning tree. Prove that a pendent edges in a connected graph G is contained in evergy spanning tree of G.

4 (a) Do as directed : **10**

(1) Given $S=\{1,2,.....10\}$ and a relation R on S where $R=\{(x,y)\backslash x+y=10\}$. What are the properties of the relation R ?

(2) Draw Hass diagram for (S_{30},D) where S_{30} is the set of divisors of 30 and D stands for relation divides.

(3) In usual notations, define $P \rightarrow Q$ write its negation also.

(4) If A = {a, b}. Write the elements of $\rho(A)$ where $\rho(A)$ is the power set of A.

(5) Define Recursion.

(b) Attempt any two : **08**

(1) Prove that the relation congruence module 5" given by $x \equiv y$ (mod 5) if and only if x–y is divisible by 5, over the set of integers, is an equivalance relation. Show that if $x_1 = y_1$ and $x_2 = y_2$ (mod 5) then $(x_1+x_2)=(y_1+y_2)$ (mod 5).

(2) Define POSET.

Prove that $<R_1 \leq>$ is a POSET.

(3) Let p and q be any two statement formulas. Construct a truth table for $\neg(p \vee q) \vee (\neg p \wedge \neg q)$

5 (a) Attempt any two : **08**

(1) Define subgroup. Show that $S \neq \varphi$ of a group G is a subgroup of (G,*) if and only if for any pair of elements $a,b \in s, a*b^{-1} \in s$

(2) Define an abelian group. Show that if every element in a group is its own inverse, then the group must be abelian.

(3) Define group homomorphism. Prove that Kernel of a homomorphism g from a group $(G,*)$ to $(H,\triangle)$ is a subgroup of $(G,*)$.

(b) Attempt any two : **08**

(1) Let $(L,\leq)$ be a lattice for any $a,b,c\in L$ prove that $b\leq c \Leftrightarrow a*b\leq a*c$

(2) Let $(L,\leq)$ be a lattice. Prove that, if a $a\leq b$ and $c\leq d$ then $a*c\leq b*d$ where $a,b,c,d\in L$ and * denotes operation of meet.

(3) Show that in a complemented, distributive lattice $a\leq b \Leftrightarrow a*b^1=0 \Leftrightarrow a^1\oplus b=1$

6 (a) Attempt any two : **08**

(1) Define a complemented lattice. Find the complements of every element of the lattice (S_{15},D).

(2) In any Boolean Algebra, show that $a=0 \Leftrightarrow ab^1+a^1b=b$

(3) Show that a chain of three or more elements is not complemented.

(b) Attempt any two : **08**

(1) Show that $f(x,y)=x^y$ is a primitive recursive functin. Also show that the function $f(x_1,x_2,y)$ define as $f(x_1,x_2,y)=\begin{cases} x_2, x_1>y \\ (x_1*y)+x_2, x_1\leq y \end{cases}$ is primitive recursive.

(2) Defined partial recursive functin. Show that the function $f(x)=\frac{x}{2}$ is a partical recursive function.

(3) Use tha Karnaugh map representation to find a minimal sum of products expression of the function $f(a,b,c,d)=\sum(0,1,2,3,13,15)$